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# The Sine Method: An Alternative Height Measurement Technique 



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#### Abstract

Height is one of the most important dimensions of trees, but few observers are fully aware of the consequences of the misapplication of conventional height measurement techniques. A new approach, the sine method, can improve height measurement by being less sensitive to the requirements of conventional techniques (similar triangles and the tangent method). We studied the sine method through a couple of comparisons. First, we demonstrated the validity of the sine method under idealized conditions by comparing tangent and sine measurements on a stationary object of a known height. Then, we compared heights collected via climbing and lowering a tape from the highest point of a number of forest-grown trees with heights measured with the sine method. The sine method offers a viable, cost effective alternative to traditional measurement approaches, especially for large or leaning trees, and for trees with broadly spreading crowns.


Keywords: Height measurement, hypsometers, similar triangles, sine method, tangent method, trigonometry.

## Introduction

Total tree height is an important measure of numerous forest conditions. Height is an indicator of the status of the tree within the population and is helpful in predicting stand development and successional patterns (Tester and others 1997, Purves and others 2008). The vertical structuring of adjacent trees largely determines the outcome of gap closure and the ability of understory species to reach the canopy (Webster and Lorimer 2005). Other functional aspects of forest ecology (e.g., water use by trees, light extinction through the canopy, wildlife habitat quality, and seed dispersal) depend partly on tree height (Boelman and others 2007, Dovciak and others 2005, Ford and Vose 2007, Parker

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and others 2002). For instance, the modeling of seed dispersal depends on the trajectory of falling seeds and the distance traveled, which are partially a function of tree height and therefore an accurate measurement of the starting height of seeds in the canopy is critical (Dovciak and others 2005, Williams and others 2006). More recently, the study of why trees grow as tall as they do has emerged as a research topic (e.g., Domec and others 2008, Nabeshima and Hiura 2008), as has metabolic scaling across size classes (Russo and others 2007).

Popular interest in "big tree" lists (e.g., American Forests 2010, Forestry Tasmania 2009) has merged with aspects of science and conservation. Researchers occasionally use champion trees to define the upper limits of species height (e.g., Botkin 1992, Bragg 2008a, Parresol 1995, Shifley and Brand 1984), and some agencies use exceptionally tall trees to help establish reserves (e.g., Forestry Tasmania 2009). Height can play a key role in developing accurate relationships between tree bole diameter and aboveground volume and productivity estimates (Newton and Amponsah 2007, Repola 2008). Certain allometric relationships are embedded within growth functions of many forest simulators - the gap models, as an example, use an increment function based in part on tree height (e.g., Botkin and others 1972, Moore 1989, Shugart 1984). Many large-scale biomass, carbon storage, and timber volume estimates are generated from diameter- and height-based equations applied to forest inventory data (e.g., Botkin and others 1993, Somogyi and others 2007). Remote sensing techniques to derive forest biomass and carbon sequestration require accurate ground-based tree height measurements for calibration and verification (Asner and others 2010, Boudreau and others 2008, Clark and others 2004, Lefsky and others 2002, Sexton and others 2009, Wang and Qi 2008).

All of these examples suggest the need for reliable quantification of tree size. Such details are not without consequence-for example, the inaccurate measurement
of tree diameter has been previously noted as a cause of overestimates of carbon storage in tropical rainforests (Clark 2002), and differences in height have potentially significant impacts on individual tree volume computations (Westfall 2008). Nevertheless, height measurement techniques are often applied without consideration for their basic assumptions. This paper introduces a new, less sensitive methodology capable of improving the accuracy and reliability of height measurement while using conventional technology.

## Mathematics behind Tree Height Determination

In addition to direct measurement with height poles or tape drops, there have been two primary methods for groundbased estimation of tree height. One approach uses the geometric principle of similar triangles (fig. 1A). This method, once incorporated in a number of hypsometers, is very sensitive to tree lean and observer error and has largely been replaced by other approaches in the forestry profession (Husch and others 2003). We will not discuss the similar triangle method (also called the "stick method") further, but note that it is still suggested by some tree measuring organizations as a means to measure champion tree heights (e.g., American Forests 2010).

The second method applies trigonometry to estimate tree height (fig. 1B). For a right triangle, the subsequent relation is always true:
$c=\tan (\alpha) b=\sin (\alpha) d$
where the length (height) of the vertical leg of this triangle (c) can be determined using the angle $\alpha$ and either the length of the horizontal leg (b) or the length of the hypotenuse (d). Most optical and electronic hypsometers adapt this relationship using the following approach:
$H T_{T A N}=\left|\tan \left(\alpha_{1}\right) b_{1}-\tan \left(\alpha_{2}\right) b_{2}\right|$
where $\mathrm{HT}_{\text {TAN }}$ is the total tree height using the tangent method, $\alpha_{1}=$ degrees of the angle between horizontal line from the observer's eye to top of the tree; $\alpha_{2}=$ degrees of the angle from horizontal line from the observer's eye to base of the tree ( $\alpha_{2}$ is negative when the base is below the eye level of the observer, thus subtracting a negative value produces a positive height component); and $\mathrm{b}_{1}=\mathrm{b}_{2}=$ the horizontal distance from eye of the observer to the midpoint of the trunk of the tree (fig. 2A). The tangent method must satisfy three main assumptions. First, the base and the actual


Figure 1-Earlier technology for measuring tree heights focused on using the geometry of similar triangles A) or the trigonometry of right triangles B).
highest point of the tree must be known. Second, the tree must be truly vertical (the highest point of the crown is exactly above the endpoint of the baseline for the horizontal plane). Third, the baselines ( $b_{1}$ and $b_{2}$ ) represent true horizontal distances. If these conditions are met, $\mathrm{HT}_{\text {TAN }}$ is an accurate and unbiased representation of total tree height.

Virtually every depiction of height measurement shows an idealized tree (usually a conifer) with a straight bole and the highest growing tip located directly above the center of the base. In reality, however, species with strong apical dominance and a pronounced leader can grow with lean (fig. 3A) or develop a branching pattern that offsets the highest top from the centerline of the bole. Many other species, especially decurrent hardwoods growing in relatively open conditions (such as the $118-\mathrm{cm}$ diameter at breast height (d.b.h.) water oak (Quercus nigra L.) in fig. 3B) develop broad, spreading crowns. For the tangent method to accurately estimate height in either of these cases, the observer first must find and compare the highest apparent extension of the crown by observing it from some distance


Figure 2-Under idealized circumstances A), tangent height and sine tree heights are exactly the same. For a truly vertical tree, this relationship does not change, regardless if the observer is located upslope B) or downslope C) from the base of the tree. The ground distance from the observer to the tree must be corrected for the slope from true horizontal to produce the actual baseline distance ( $\mathrm{b}_{\mathrm{c}}$ ).
away, then find the nadir beneath that exact same point through an often leafy and branch-filled crown, and finally must determine the distance between the observation point and the nadir (the baseline length). The tangent method involves considerable effort when there are multiple possible high points in a large, spreading crown (fig. 3B) to evaluate - to ensure total height is found, several different spots must be tested with the tangent method before the maximum value can be determined. Furthermore, other attributes of a tree may influence the accuracy of height measurement, such as its growth on a steep slope with lean, particularly when it has an offset growing tip (fig. 4A). Even straight trees with thick boles can produce offset errors if the highest tip does not lie directly above the surface of the stem where the baseline distance is measured, or if the baseline distance is not adjusted to reflect the actual configuration (fig. 4B).

Uneven terrain adds another level of complexity to the tangent method. Prior to self-correcting electronic hypsometers, the observer would need to manually adjust baseline length to reflect the true horizontal distance. On sloping ground, regardless of the viewing angle (figs. 2B and 2C), the observer must make a trigonometric conversion between the uncorrected (slope) distance measured with their tape $\left(\mathrm{b}_{\mathrm{u}}\right)$ and the true horizontal baseline distance. Using a tape and clinometer, one possible adjustment is:
$b_{c}=\cos \left(\alpha_{u}\right) b_{u}$
where the angle $\left(\alpha_{u}\right)$ is to the point on the stem (u) where the tape is affixed. This correction assumes that the tape is stretched tight and held parallel to the slope between the observer and the tree. After this adjustment, the corrected baseline distance $\mathrm{b}_{\mathrm{c}}=\mathrm{b}_{1}=\mathrm{b}_{2}$, and the tangent height can be determined using $b_{c}$ and the angles to the top and bottom of the tree $\left(\alpha_{1}\right.$ and $\left.\alpha_{2}\right)$. Figures 2B and 2C illustrate the need to correct the baseline on very steep slopes, although the effects of slope are noticeable on even gentle grades and should be addressed.

For decades, the standard instruments for measuring tree height have been similar triangle-based mechanical hypsometers or hand-held clinometers and measuring tapes. These tools are still popular because they are inexpensive, easy to learn, and quick to apply in the field, but they are gradually being replaced by newer technologies. All commercially available electronic (laser- or sound-based) hypsometers measure at least slope distance. Those with integrated clinometers can reduce tangent-based height estimation errors by automatically correcting baseline length



Figure 3-The primary source of error with the tangent method arises from the failure to correct for the proper baseline length (i.e., $b_{2}$ is used rather than $b_{1}$ ), as may happen with a leaning tree A). This error also appears with large crowns B), especially when rounded or flat-topped, as few observers actually adjust their baseline length when using the tangent method to the point directly below the crown high point. In this example, each of the arrows indicate potential high spots of the crown that would need to have their baseline distances calculated after translating these through the thick crown to their respective nadirs. With the sine method and a laser hypsometer with a continuous scanning mode, it was possible to check all of these points in seconds from the same observation spot to find the true height of this water oak (dashed circle).


Figure 4-Horizontal offset errors arise when the tangent method is not corrected for differences between the appropriate baseline lengths. If the same baseline length is used $\left(b_{1}=b_{2}\right)$ on a leaning tree without correction $\left.A\right)$, the observer will generate horizontal offset errors on both the upper and lower triangles. Horizontal offset can also occur when the highest point on the crown is displaced away from the bole B), again requiring two different baseline distances placed at the appropriate spacing from the observer in order to calculate the correct height.
for vertical trees on uneven ground by converting slope distance into horizontal distance (figs. 2B and 2C).

Prior to the advent of reasonably priced electronic rangefinders, it was not practical to measure the slope distance $\left(\mathrm{d}_{1}\right)$ from the observer's eye to the top of the tree (measuring slope distance to the base of the stem $\left(\mathrm{d}_{2}\right)$ directly with a tape is straightforward). Once this technology became available, a new approach to triangulation became feasible:
$H T_{S I N}=\left|\sin \left(\alpha_{1}\right) d_{1}-\sin \left(\alpha_{2}\right) d_{2}\right|$
where $\mathrm{HT}_{\text {SIN }}$ is the height calculated using the sine method. When measuring the height of a perfectly vertical object, $\mathrm{HT}_{\text {TAN }}$ should equal $\mathrm{HT}_{\text {SIN }}$ (fig. 2A). The sine method yields the vertical distance between parallel horizontal planes, one touching the base of the tree and the other touching the highest point of the tree (fig. 3A). Because of this trigonometry, the sine method does not require the tree to be straight, the high point of the tree to be directly above the trunk, or the ground to be level (Blozan 2006, Bragg 2008b). The sine method is a realization of the hemispherical approach of Grosenbaugh (1980), who recognized the potential biases of previous technology that assumed trees were truly vertical.

## Demonstrating the Sine Method

We demonstrated and validated the sine method by measuring a fixed object of known height with a stationary distance-measuring device (sensu Bruce 1975). For this test, 21 measurements were taken with an Impulse ${ }^{\circledR}$ 200LR laser rangefinder on a clock tower on the University of ArkansasMonticello (UAM) campus from distances of just under 15 m to over 165 m . The Impulse 200LR was affixed to a stable tripod and the magnified red-dot scope used to locate, target, and measure the pointed top and level base of the clock tower, with a distance-measuring accuracy of 3 to 5 cm and an inclination accuracy of 0.1 degrees (up to 500 m from the target; Laser Technology, Inc. (2010)). We compared the heights determined by the tangent and sine methods with a paired t-test.

Table 1 provides the data collected when both methods were used to estimate the height of a $12.52-\mathrm{m}$-tall clock tower on the UAM campus. The mean difference for 21 measurements of this tower using the traditional and new methods was a statistically insignificant $0.03 \mathrm{~m}(12.51$ m (tangent) versus 12.48 m (sine), paired- $\mathrm{t}=0.88, \mathrm{P}=$ 0.3908 ), verifying that $\mathrm{HT}_{\text {TAN }}=\mathrm{HT}_{\text {SIN }}$ for vertical objects. Each technique had low variance on this structure, with standard deviations of 0.14 and 0.12 m for the tangent and
sine methods, respectively, within the given measurement error of the device ( $\pm 0.05 \mathrm{~m}$ ) and observer error (e.g., inconsistent measurement locations). Both techniques performed well, even though the ground surrounding the tower was not entirely level, as the laser was stationed at elevations from 0.5 to 4.5 m above the base of the clock tower. High precision and accuracy of the measurements of this tower were expected, though, as it is located in a part of campus with few obstructions and no extraneous factors (e.g., wind, lighting, intervening vegetation) affected the view.

To provide further evidence of the accuracy of the sine method, we also compared directly measured heights of climbed trees with sine-based measurements. Forty-two large, forest-grown trees were climbed and their height determined by lowering a measuring tape from the highest climbable point of the crown to the ground (usually, a pole of known length was extended to get to the very top of the tree). The sine method was then used to calculate height of the climbed trees using the exact same point in the crown. The measurements for this effort were taken with TruPulse ${ }^{\circledR}$ 200 or TruPulse ${ }^{\circledR} 360$ hand-held laser hypsometers with manufacturer-stated distance accuracies of $\pm 0.3 \mathrm{~m}$ (out to $1,000 \mathrm{~m}$ ) and electronic clinometer accuracy of $\pm 0.25$ degrees (Laser Technology, Inc. 2010) or recreation-grade laser rangefinders (e.g., Nikon ProStaff® 440) that are less exact ( $\pm 1 \mathrm{~m}$ distance accuracy) and produce only slope distances (angle readings must be taken with a separate clinometer).

Tape drop heights of 42 forest-grown trees (table 2) deviated from heights measured by the sine method by an average of 0.01 m (range: -0.91 to 0.61 m , standard deviation $=$ 0.31 m ), with an average relative error of 0.03 percent (range: -1.92 to 1.39 percent, standard deviation $=0.64$ percent). Hence, with the accurate laser rangefinders and electronic clinometers available today, instrument error when measuring total tree heights with the sine method can be expected to be consistently less than 1 percent for experienced users.

## Comparing the Tangent and Sine Methods

For trees on level ground with straight boles, evenly distributed crowns, and a distinct leader located directly above the base of the stem, there are not likely to be any significant differences between either height measurement technique (fig. 2A). This condition exists in nature, especially in young conifer stands, but is likely the exception rather than the rule in older stands where most trees have broad crowns with several leaders nearly equal in
height above the base of the tree. Large, flat-crowned trees tend to produce height overestimates (Belyea 1931, Husch and others 2003) with the tangent method. Under these circumstances, if the proper top is not identified and the correct baseline distance calculated for the tangent method, total height estimates under these circumstances "....are of little value" (Husch and others 2003: 109). Another challenge for the tangent method is the need to distinguish between the true high point of a crown and subordinate branches projecting towards or away from the observer. Similar to the leaning tree in figure 3 , without correction of baseline length any subordinate branches facing the observer will thus appear to be taller than what they actually are, and branches extending away from the observer may seem shorter.

Even if the point measured is a subordinate branch, the sine method will only underestimate total height. This bias can be ameliorated, if not completely eliminated, by using
the scanning mode found on most laser rangefinders to search for other points higher than the initial observation. Implementing this crown-scanning approach from multiple viewing angles, then taking the maximum of this collection of sine heights, is the best means to estimate true total height short of direct measurement. Without locating the nadir of numerous high points on the ground through the crown and then measuring the length of a series of corresponding baselines, it is virtually impossible to comparably assess all likely high points in a crown with the tangent method (fig. 3B). The choice of the highest apparent point has been instilled on measurers from the beginning with the tangent method - with a fixed baseline, increasing the angle of inclination is the only way to maximize the total height of a given tree. For most observers using this approach, selecting an apparently lower point in the crown might seem counterintuitive, especially given the uncertainty that this particular adjustment will yield the highest tip.

Table 1-Tangent and sine height estimates for the clock tower on the University of Arkansas-Monticello campus (from the base of the tower to the point on the top)

| Direction | Horizontal distance | Tangent height estimate | Sine height estimate | Laser height above tower base |
| :---: | :---: | :---: | :---: | :---: |
| NW | 23.25 | 1232 | 1230 | 1.10 |
| NW | 50.46 | 12.55 | 12.39 | 1.49 |
| NW | 69.19 | 12.64 | 12.36 | 2.18 |
| NW | 111.63 | 12.46 | 12.58 | 2.51 |
| NE | 18.20 | 12.28 | 12.13 | 1.03 |
| NE | 39.64 | 12.65 | 12.64 | 0.96 |
| NE | 58.67 | 12.39 | 12.46 | 1.00 |
| NE | 85.77 | 12.45 | 12.46 | 0.72 |
| NE | 166.10 | 12.28 | 12.62 | 0.48 |
| SW | 14.86 | 12.45 | 12.45 | 2.05 |
| SW | 21.98 | 12.44 | 12.45 | 2.27 |
| SW | 34.16 | 12.78 | 12.57 | 2.83 |
| SW | 47.79 | 12.40 | 12.62 | 3.55 |
| SW | 62.55 | 12.70 | 12.44 | 4.54 |
| SE | 22.15 | 12.67 | 12.53 | 2.23 |
| SE | 39.15 | 12.69 | 12.53 | 2.91 |
| SE | 59.01 | 12.45 | 12.53 | 3.51 |
| E | 24.57 | 12.51 | 12.45 | 1.58 |
| E | 44.26 | 12.59 | 12.50 | 1.78 |
| E | 64.56 | 12.45 | 12.54 | 1.90 |
| E | 91.45 | 12.51 | 12.49 | 1.99 |
| Mean | 54.73 | 12.51 | 12.48 | 2.03 |
| Standard deviation | 36.32 | 0.14 | 0.12 | 1.03 |

Note: Direct measurement of the clock tower using a height pole yielded a height of 12.52 m .

Table 2-Comparison of sine height with direct measurements (tape drop) for 42 large trees

| Species | Sine height | Tape drop height | Difference | Relative error |
| :---: | :---: | :---: | :---: | :---: |
|  |  | -- m |  | - percent - |
| Eastern hemlock | 52.64 | 52.76 | -0.12 | -0.23 |
| Eastern white pine | 52.73 | 52.73 | 0.00 | 0.00 |
| Eastern hemlock | 51.94 | 52.46 | -0.52 | -0.99 |
| Eastern hemlock | 52.30 | 52.33 | -0.03 | -0.06 |
| Eastern hemlock | 52.49 | 52.27 | 0.21 | 0.41 |
| Eastern hemlock | 51.42 | 51.63 | -0.21 | -0.41 |
| Eastern hemlock | 51.48 | 51.48 | 0.00 | 0.00 |
| Eastern hemlock | 50.51 | 51.05 | -0.55 | -1.07 |
| Eastern hemlock | 51.18 | 50.99 | 0.18 | 0.36 |
| Eastern hemlock | 51.02 | 50.96 | 0.06 | 0.12 |
| Loblolly pine | 50.72 | 50.90 | -0.18 | -0.36 |
| Eastern hemlock | 50.90 | 50.81 | 0.09 | 0.18 |
| Eastern hemlock | 50.84 | 50.63 | 0.21 | 0.42 |
| Eastern hemlock | 50.69 | 50.63 | 0.06 | 0.12 |
| Eastern white pine | 49.87 | 50.29 | -0.43 | -0.85 |
| Eastern hemlock | 50.29 | 50.20 | 0.09 | 0.18 |
| Eastern white pine | 50.63 | 50.17 | 0.46 | 0.91 |
| Eastern white pine | 49.83 | 49.80 | 0.03 | 0.06 |
| Eastern hemlock | 49.44 | 49.32 | 0.12 | 0.25 |
| Eastern white pine | 49.38 | 49.04 | 0.34 | 0.68 |
| Eastern hemlock | 48.65 | 49.01 | -0.37 | -0.75 |
| Eastern white pine | 48.46 | 48.86 | -0.40 | -0.81 |
| Eastern white pine | 49.23 | 48.83 | 0.40 | 0.81 |
| Eastern white pine | 49.23 | 48.65 | 0.58 | 1.19 |
| Eastern hemlock | 48.74 | 48.65 | 0.09 | 0.19 |
| Eastern hemlock | 47.85 | 48.13 | -0.27 | -0.57 |
| Eastern hemlock | 46.73 | 47.64 | -0.91 | -1.92 |
| Eastern white pine | 47.43 | 47.24 | 0.18 | 0.39 |
| Eastern hemlock | 47.18 | 47.03 | 0.15 | 0.32 |
| Eastern hemlock | 46.60 | 46.70 | -0.09 | -0.20 |
| Eastern hemlock | 46.02 | 46.18 | -0.15 | -0.33 |
| Eastern hemlock | 45.99 | 45.75 | 0.24 | 0.53 |
| Eastern hemlock | 44.41 | 44.74 | -0.34 | -0.75 |
| Eastern white pine | 44.65 | 44.68 | -0.03 | -0.07 |
| Eastern hemlock | 44.38 | 44.32 | 0.06 | 0.14 |
| Eastern hemlock | 44.50 | 43.89 | 0.61 | 1.39 |
| Eastern hemlock | 43.92 | 43.77 | 0.15 | 0.35 |
| Eastern white pine | 44.04 | 43.68 | 0.37 | 0.84 |
| Eastern hemlock | 43.89 | 43.68 | 0.21 | 0.49 |
| Eastern hemlock | 36.67 | 36.67 | 0.00 | 0.00 |
| Eastern hemlock | 35.36 | 35.20 | 0.15 | 0.43 |
| Southern red oak | 28.96 | 28.96 | 0.00 | 0.00 |
| Mean | 47.70 | 47.68 | 0.01 | 0.03 |
| Standard deviation | 4.85 | 4.88 | 0.31 | 0.64 |

The sine method can be challenging under conditions when the canopy or the understory are dense, as the observer must have an unobstructed view of the high point and base of the tree. Most laser hypsometers have a fairly narrow beam that allows them to penetrate crowns, or can be adjusted with either electronic distance "gates" or via the use of special reflectors to minimize the influence of intervening vegetation. It is also possible to hybridize the sine and tangent methods to facilitate measurement in dense understories and midcanopies. For instance, on flat to gently sloping ground for relatively straight trees, one can use the following approximation:
$H T=\left|\sin \left(\alpha_{1}\right) d_{1}-\tan \left(\alpha_{2}\right) d_{2}\right|$

The top height component ( $\alpha_{1}$ and $d_{1}$ ) should still be done using the sine method, as this is where the greatest potential for error occurs, but the bottom height (from horizontal to the base of the stem) rarely differs between the sine and tangent methods unless the ground slopes steeply or the tree has an extreme lean.

The biggest advantage to the sine method is that it eliminates the most problematic assumptions of the tangent method encountered in the field, and therefore substantially increases overall measurement accuracy. Violating the assumptions of the similar triangle or tangent methods can produce spectacularly large errors in tree height, especially when measurements are taken in close proximity to the stem. There are numerous examples of champion trees that have been re-measured with the sine method only to find that the similar triangle- or tangent-based errors exceed 15 m (Eastern Native Tree Society 2009). For example, a former national champion bitternut hickory (Carya cordiformis (Wangenh.) K. Koch) in western North Carolina was first reported at 57.9 m with the tangent method and later re-measured by the sine method at 37.5 m . Likewise, a red maple (Acer rubrum L.) from Michigan originally measured at 54.6 m was eventually measured at 36.6 m with the sine method. In both cases, these trees did not have their tops killed or broken between the two measurements, but rather the observers failed to correctly apply conventional height measurement techniques.

The sine method is sometimes more time-consuming to apply because it requires that the laser beam directly strikes the object being measured. However, the sine method is a reliable and useful alternative to the tangent method because of the freedom it offers to observe and measure the object from any direction and distance without concern for tree lean or crown displacement. We believe few observers spend the time needed to follow the textbook corrective procedures
suggested for the tangent method. Rather, to minimize the time and expense of collecting the most accurate height data possible, most people, it appears, tend to only move to a point that appears to be perpendicular to the lean, and then use that angle and distance to the stem to estimate height. Such an ad hoc correction is incapable of measuring the three-dimensional complexity of most tree crowns, and when combined with the observer's innate tendency to then select the highest apparent point when determining total tree height, is likely to overestimate this metric. Also, in a further attempt to reduce height sampling time, it is our experience that observers often measure several trees from the same viewpoint, regardless of their lean or crown structure. This tendency has been seen from the first distance-independent hypsometers (e.g., Krauch 1918) and is likely to be even more common with horizontal baseline-correcting electronic hypsometers, since the user may incorrectly believe the device corrects for all sources of measurement error.

## Conclusions

The sine method provides direct (not extrapolated) measurements of observed points on the tree, thereby generating tree height as the elevation difference between two horizontal planes. This geometric translation of a threedimensional object thereby eliminates the need to conduct ad hoc adjustments for tree lean, offset crown high points, and ground slope in the field. When properly applied with modern laser technology, the sine method should prove no more onerous to measure than current techniques, and it is compatible with all accurately measured tree heights from past inventories. This technique has been used to correct some overestimated tree height values from champion tree data which had been previously cited by silvicultural and ecological texts as authoritative. Whether or not the sine method will supplant current approaches to measuring tree height has yet to be determined-however, its accuracy, reliability, and repeatability suggest that it can be considered a standard for any science-based studies of forest conditions that include height as a parameter.

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